Radicals:

1. $\sqrt[4]{243} = \sqrt[4]{3^5} = 3\sqrt[4]{3}$

2. $\sqrt[3]{-125} = -\sqrt[3]{5^3} = -5$

3. $\sqrt[5]{(x+2)^7} = (x+2)\sqrt[5]{(x+2)^2}$

4. $x^{\frac{9}{4}} = \sqrt[4]{x^9} = x^2\sqrt[4]{x}$

5. $(x^7 y^{11})^{\frac{1}{3}} = \sqrt[3]{x^7 y^{11}} = x^2 y^3 \sqrt[3]{xy^2}$

6. $(3x+4)^{\frac{7}{3}} = \sqrt[3]{(3x+4)^7} = (3x+4)^2 \sqrt[3]{(3x+4)}$

7. $\sqrt[4]{32x^7} = \sqrt[4]{2^5 x^7} = 2x\sqrt[4]{2x^3}$

8. $\sqrt[5]{160x^{24}y^{11}} = \sqrt[5]{2^5 \cdot 5x^{24}y^{11}} = 2x^4 y^2 \sqrt[5]{5x^4 y}$

9. $\sqrt[6]{x^3} = \left(x^3\right)^{\frac{1}{6}} = x^{\frac{3}{6}} = x^{\frac{1}{2}} = \sqrt{x}$

10. $\sqrt[12]{x^6} = \left(x^6\right)^{\frac{1}{12}} = x^{\frac{6}{12}} = x^{\frac{1}{2}} = \sqrt{x}$

11. $5\sqrt{x} + 7\sqrt{x} = 12\sqrt{x}$

12. $6\sqrt[7]{x} - 3\sqrt[7]{x} + \sqrt[7]{x} = 4\sqrt[7]{x}$

13. $7\sqrt[3]{x} - 2\sqrt[6]{x^2} = 7\sqrt[3]{x} - 2\sqrt[3]{x} = 5\sqrt[3]{x}$

14. $3\sqrt[4]{x} + 6\sqrt{x} - 3(2\sqrt[4]{x} - 5\sqrt{x}) = 3\sqrt[4]{x} + 6\sqrt{x} - 6\sqrt[4]{x} + 15\sqrt{x} = -3\sqrt[4]{x} + 21\sqrt{x}$

15. $3\sqrt{8x} - 4\sqrt{2x} = 3\sqrt{2^3 x} - 4\sqrt{2x} = 3\cdot 2\sqrt{2x} - 4\sqrt{2x} = 2\sqrt{2x}$

16. $\begin{aligned} &-2\sqrt[3]{16x^4} + 5x\sqrt[3]{54x} = -2\sqrt[3]{2^4 x^4} + 5x\sqrt[3]{2\cdot 3^3 x} = -2\cdot 2x\sqrt[3]{2x} + 5\cdot 3x\sqrt[3]{2x} = \\ &-4x\sqrt[3]{2x} + 15x\sqrt[3]{2x} = 11x\sqrt[3]{2x} \end{aligned}$

17. $2\sqrt{5x}\left(4\sqrt{5x} - 2\right) = 8\cdot 5x - 4\sqrt{5x} = 40x - 4\sqrt{5x}$

18. $4\sqrt{3x}\left(2\sqrt{x} + 5\sqrt{3}\right) = 8\sqrt{3x^2} + 30\sqrt{3^2 x} = 8x\sqrt{3} + 30\cdot 3\sqrt{x} = 8x\sqrt{3} + 90\sqrt{x}$

19. $\left(2\sqrt{x} - 3\right)\left(2\sqrt{x} + 3\right) = 4\sqrt{x^2} + 6\sqrt{x} - 6\sqrt{x} - 9 = 4x - 9$

20. $\left(5\sqrt{x} + 2\right)\left(3\sqrt{x} - 1\right) = 15\sqrt{x^2} - 5\sqrt{x} + 6\sqrt{x} - 2 = 15x + \sqrt{x} - 2$

21. $\left(-3\sqrt{x}\right)\sqrt[3]{x} = -3x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} = -3x^{\frac{3}{6}} \cdot x^{\frac{2}{6}} = -3x^{\frac{5}{6}} = -3\sqrt[6]{x^5}$

22. $\sqrt{xy}\cdot\sqrt[3]{x^2y}\cdot\sqrt[4]{x^3y^5}=(xy)^{\frac{1}{2}}(x^2y)^{\frac{1}{3}}(x^3y^5)^{\frac{1}{4}}=x^{\frac{1}{2}}y^{\frac{1}{2}}x^{\frac{2}{3}}y^{\frac{1}{3}}x^{\frac{3}{4}}y^{\frac{5}{4}}=x^{\frac{6}{12}}y^{\frac{6}{12}}x^{\frac{8}{12}}y^{\frac{4}{12}}x^{\frac{9}{12}}y^{\frac{15}{12}}=$

$x^{\frac{23}{12}}y^{\frac{25}{12}}=\sqrt[12]{x^{23}y^{25}}=xy^2\sqrt[12]{x^{11}y}$

23. $\dfrac{5}{7\sqrt{x}}=\dfrac{5}{7\sqrt{x}}\cdot\dfrac{\sqrt{x}}{\sqrt{x}}=\dfrac{5\sqrt{x}}{7x}$

24. $\dfrac{2}{\sqrt{8x}}=\dfrac{2}{\sqrt{2^3x}}\cdot\dfrac{\sqrt{2x}}{\sqrt{2x}}=\dfrac{2\sqrt{2x}}{2^2x}=\dfrac{\sqrt{2x}}{2x}$

25. $\dfrac{2}{\sqrt[5]{x^3y^2}}=\dfrac{2}{\sqrt[5]{x^3y^2}}\cdot\dfrac{\sqrt[5]{x^2y^3}}{\sqrt[5]{x^2y^3}}=\dfrac{2\sqrt[5]{x^2y^3}}{xy}$

26. $\dfrac{6}{\sqrt[4]{32x^9y^{13}}}=\dfrac{6}{\sqrt[4]{2^5x^9y^{13}}}\cdot\dfrac{\sqrt[4]{2^3x^3y^3}}{\sqrt[4]{2^3x^3y^3}}=\dfrac{6\sqrt[4]{2^3x^3y^3}}{2^2x^3y^4}=\dfrac{3\sqrt[4]{2^3x^3y^3}}{2x^3y^4}$

27. $\sqrt[4]{\dfrac{7}{4x^3}}=\sqrt[4]{\dfrac{7}{2^2x^3}}=\dfrac{\sqrt[4]{7}}{\sqrt[4]{2^2x^3}}\cdot\dfrac{\sqrt[4]{2^2x}}{\sqrt[4]{2^2x}}=\dfrac{\sqrt[4]{2^2\cdot7x}}{2x}$

28. $\dfrac{\sqrt[3]{25xy^2}\sqrt{5xy}}{\sqrt[4]{125x^3y^{-2}}}=\dfrac{\left(5^2xy^2\right)^{\frac{1}{3}}(5xy)^{\frac{1}{2}}}{\left(5^3x^3y^{-2}\right)^{\frac{1}{4}}}=\dfrac{5^{\frac{2}{3}}x^{\frac{1}{3}}y^{\frac{2}{3}}5^{\frac{1}{2}}x^{\frac{1}{2}}y^{\frac{1}{2}}}{5^{\frac{3}{4}}x^{\frac{3}{4}}y^{\frac{-2}{4}}}=\dfrac{5^{\frac{8}{12}}x^{\frac{4}{12}}y^{\frac{8}{12}}5^{\frac{6}{12}}x^{\frac{6}{12}}y^{\frac{6}{12}}}{5^{\frac{9}{12}}x^{\frac{9}{12}}y^{\frac{-8}{12}}}=5^{\frac{5}{12}}x^{\frac{1}{12}}y^{\frac{22}{12}}=$

$\sqrt[12]{5^5xy^{22}}=y\sqrt[12]{5^5xy^{10}}$

29. $\dfrac{5}{\sqrt{x}-5}=\dfrac{5}{\left(\sqrt{x}-5\right)}\cdot\dfrac{\left(\sqrt{x}+5\right)}{\left(\sqrt{x}+5\right)}=\dfrac{5\sqrt{x}+25}{x-25}$

30. $\dfrac{\sqrt{x}-2}{3\sqrt{x}+1}=\dfrac{\left(\sqrt{x}-2\right)}{\left(3\sqrt{x}+1\right)}\cdot\dfrac{\left(3\sqrt{x}-1\right)}{\left(3\sqrt{x}-1\right)}=\dfrac{3x-\sqrt{x}-6\sqrt{x}+2}{9x-1}=\dfrac{3x-7\sqrt{x}+2}{9x-1}$

31. $\sqrt{x}-5=0\Rightarrow\sqrt{x}=5\Rightarrow\left(\sqrt{x}\right)^2=(5)^2\Rightarrow x=25$

32. $3\sqrt{x}+21=4 \Rightarrow 3\sqrt{x}=-17 \Rightarrow \left(3\sqrt{x}\right)^2=\left(-17\right)^2 \Rightarrow 9x=289 \Rightarrow x=\dfrac{289}{9}$

33. $2\sqrt{x}-12=-7\sqrt{x}+16 \Rightarrow 9\sqrt{x}=28 \Rightarrow \left(9\sqrt{x}\right)^2=\left(28\right)^2 \Rightarrow 81z=784 \Rightarrow z=\dfrac{784}{81}$

34. $\sqrt{13+4x}=x+4 \Rightarrow \left(\sqrt{13+4x}\right)^2=\left(x+4\right)^2 \Rightarrow 13+4x=x^2+8x+16 \Rightarrow 0=x^2+4x+3 \Rightarrow$
$0=(x+3)(x+1) \Rightarrow x=-3, x=-1$

35. $\sqrt{3x+4}-\sqrt{2x-4}=2 \Rightarrow \left(\sqrt{3x+4}\right)^2=\left(\sqrt{2x-4}+2\right)^2 \Rightarrow 3x+4=2x-4+4\sqrt{2x-4}+4 \Rightarrow$
$x+4=4\sqrt{2x-4} \Rightarrow \left(x+4\right)^2=\left(4\sqrt{2x-4}\right)^2 \Rightarrow x^2+8x+16=16(2x-4) \Rightarrow$
$x^2+8x+16=32x-64 \Rightarrow x^2-24x+80=0 \Rightarrow (x-20)(x-4)=0 \Rightarrow x=20, x=4$

36. $\sqrt{\sqrt{x}+1}=3 \Rightarrow \left(\sqrt{\sqrt{x}+1}\right)^2=\left(3\right)^2 \Rightarrow \sqrt{x}+1=9 \Rightarrow \sqrt{x}=8 \Rightarrow \left(\sqrt{x}\right)^2=\left(8\right)^2 \Rightarrow x=64$